Physics 798I
Superconductivity
Fall 2012
Homework 1
Due Wednesday, September 12, 2012

Homework Policy

Your grade will be based on homework and a paper. In exchange for not giving exams, I ask that you do the homework. You may work on homework together, but not doing the homework will imperil your grade--I am willing to give bad grades if homework is not done.

Please hand in your homework on time. I will not accept late homework, unless a valid excuse (such as illness) is given, preferably before the homework is due.

1. **Screening in a superconducting slab** (this is essentially problem 3.2 of Annett). Solve the London equations for an infinite superconducting plate of finite thickness 2t, assuming the magnetic field B_0 is applied parallel to both surfaces. Find both the magnetic field and the supercurrent inside the slab. As examples, plot the current and magnetic field for a thickness $2t = \lambda$, and $2t = 10\lambda$.

2. **Two-fluid model**. A more realistic model for a superconductor assumes that there is a density n_n of normal electrons which obey a Drude-like equation

$$\frac{dJ_n}{dt} = \frac{n_n q^2}{m} E - \frac{J_n}{\tau}$$

as well as a density n_s of superelectrons which obey a London equation

$$\frac{dJ_s}{dt} = \frac{n_s q^2}{m} E$$

a) Using the $e^{+i\omega t}$ time convention, find the frequency-dependent complex conductivity $\sigma(\omega)$. Assume that each "fluid" responds independently to the electric field.
b) What simple lumped-element circuit has an admittance Y=1/Z with the same frequency dependence?
c) Show that, in the low-frequency limit, the normal-fluid response is purely ohmic, while the superfluid response is purely inductive. In this limit, plot $\sigma_1(T)$ and $\sigma_2(T)$ vs T using the empirical relationships

$$n_s(T) = n_o\left[1 - \left(\frac{T}{T_c}\right)^4\right] \quad ; \quad n_n(T) = n_o - n_s(T)$$

where n_o is the density of electrons in the material. The expression for $n_s(T)$ is a fairly good approximation for the superfluid density in a clean metal, but the second expression is seriously flawed: $n_s(T) + n_n(T)$ is not equal to the total electron density.